

PP36717. Proposed by Pirkuliyev Rovsen.

Prove without softs:

$$\frac{2+e^2}{2+\pi^2} > \sqrt[3]{e^{2(e-\pi)}}.$$

Solution by Arkady Alt, San Jose, California, USA.

Let $h(x) := \frac{e^{\frac{2x}{3}}}{2+x^2}$. Since $h'(x) = \frac{2e^{\frac{2x}{3}}(x-1)(x-2)}{3(x^2+2)^2} > 0$ then $h(x)$ increase in $(2, \infty)$

and, therefore, $h(\pi) > h(e) \Leftrightarrow \frac{e^{\frac{2\pi}{3}}}{2+\pi^2} > \frac{e^{\frac{2e}{3}}}{2+e^2} \Leftrightarrow \frac{2+e^2}{2+\pi^2} > e^{\frac{2(e-\pi)}{3}}$.